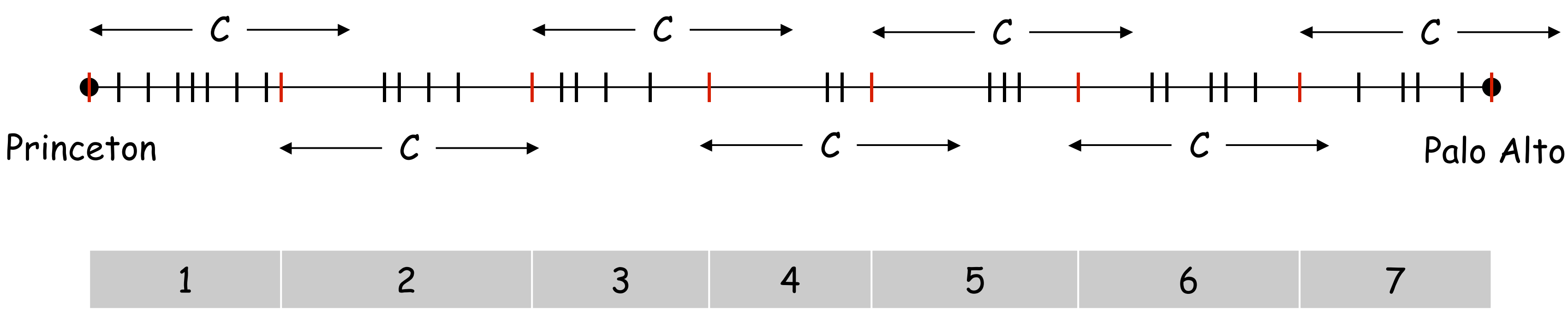
Selecting Breakpoints

Selecting breakpoints.

- Road trip from Princeton to Palo Alto along fixed route.
- Refueling stations at certain points along the way.
- Fuel capacity = C.
- Goal: makes as few refueling stops as possible.

Greedy algorithm. Go as far as you can before refueling.



Selecting Breakpoints: Greedy Algorithm

Truck driver's algorithm.

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Sort breakpoints so that: 0 = b0 < b1 < b2 < ... < bn = L

S ← {0}   ← breakpoints selected
x ← 0     ← current location

while (x ≠ bn)
   let p be largest integer such that bp ≤ x + C
   if (bp = x)
      return "no solution"
   x ← bp
   S ← S ∪ {p}
return S
```

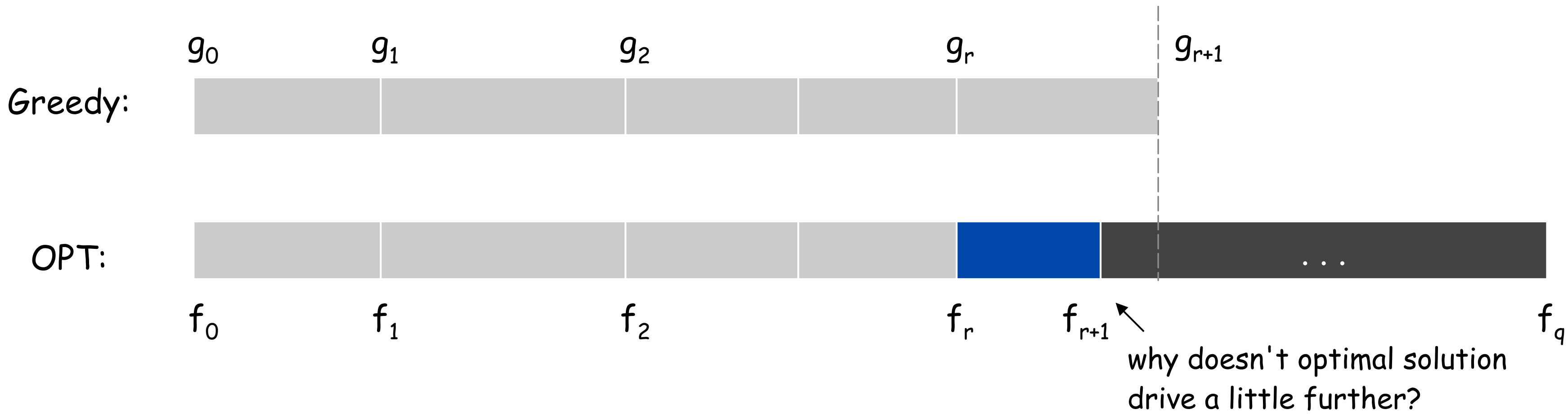
Implementation. $O(n \log n)$
- Use binary search to select each breakpoint p.

Selecting Breakpoints: Correctness

Theorem. Greedy algorithm is optimal.

Pf. (by contradiction)

- Assume greedy is not optimal, and let's see what happens.
- Let $0 = g_0 < g_1 < \ldots < g_p = L$ denote set of breakpoints chosen by greedy.
- Let $0 = f_0 < f_1 < \ldots < f_q = L$ denote set of breakpoints in an optimal solution with $f_0 = g_0$, $f_1 = g_1$, . . . , $f_r = g_r$ for largest possible value of r.
- Note: $g_{r+1} > f_{r+1}$ by greedy choice of algorithm.



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